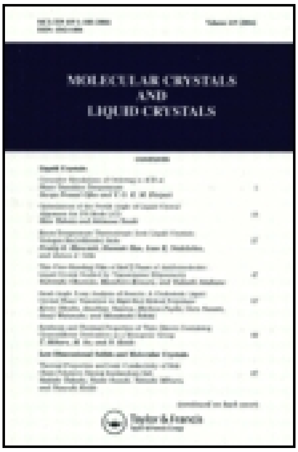

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FLOW AND BACKFLOW EFFECTS IN A CONTINUUM THEORY FOR SMECTIC *C* LIQUID CRYSTALS

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Abstract This paper discusses some flow problems for smectic *C* liquid crystals based on a continuum theory proposed by Leslie, Stewart and Nakagawa. Analyses are presented of certain flow instabilities that can occur, and also of a backflow effect that follows the removal of an external field.

1. INTRODUCTION

In a recent paper Leslie, Stewart and Nakagawa[1] propose a theory for smectic *C* liquid crystals that may prove useful for the study of certain phenomena in these layered anisotropic liquids. Their theory employs two simplifying assumptions that clearly restrict its range of applicability. These are that the layers although deformable remain of constant thickness, and also that the tilt of the alignment with respect to the layer normal does not change. The latter certainly appears reasonable when pretransitional and thermal effects are insignificant, but the former clearly renders the theory inappropriate in a number of situations.

The theory employs two unit vectors to describe smectic *C* configurations. The first a density wave vector that here simply reduces to the unit layer normal **a**, and the second a unit vector **c** orthogonal to **a** describing the direction of tilt with respect to the layer normal. The static version of the theory employs an energy quadratic in the gradients of these two vectors that contains nine bulk terms for non-chiral materials[2], and eleven for chiral smectic *C* liquid crystals.[3] While this represents an increase in complexity over nematic theory, it remains manageable and indeed progress has been possible in analysing Dupin[4] and parabolic cyclides.[5]

However, the dynamic version of the theory includes in addition twenty viscous coefficients which in many respects seems excessive, although this is necessary to model flow alignment when planar layers slide over one another.

Notwithstanding the apparent complexity of dynamic theory, it has proved possible to analyse certain effects in shear flow[6,7], particularly for the case when the layers are parallel to the bounding plates. In this paper we consider some other flow problems in this geometry, and show that in principle a number of effects are possible that allow further comparison between theory and experiment. These include simple flow instabilities and a backflow effect, both rather similar to corresponding effects that occur in nematics.[8,9] The degree to which predictions and observations agree is of course a test of the theory, but perhaps just as important the relative scale of these effects provides some measure of parameters in the theory, indicating their relative importance.

The problems discussed do point to one general conclusion, namely that changes in alignment in general induce flow, and an experimental measure of this flow of course gives some indication of the magnitudes of the associated viscosities, although this must be approached with some caution given certain simplifying assumptions employed in our analysis. For example, we assume strong anchoring at the plates, but this may not be the most appropriate boundary condition for the geometry discussed. However, it does seem premature to tackle more complex analyses until some experimental evidence is available.

2. CONTINUUM THEORY

This section presents a brief summary of the continuum equations proposed by Leslie, Stewart and Nakagawa[1] to model certain macroscopic behaviour of smectic C liquid crystals, in which variations of tilt of the alignment with respect to the layer normal and of layer thickness may prove negligible. The theory employs a unit vector **a** normal to the layers to describe the smectic layering, with the direction of tilt given by a second orthogonal unit vector **c**. These vectors are therefore subject to the constraints

$$\mathbf{a}.\mathbf{a} = \mathbf{c}.\mathbf{c} = 1, \quad \mathbf{a}.\mathbf{c} = 0 \quad , \quad \mathrm{curl}\,\mathbf{a} = 0, \tag{2.1}$$

the latter a consequence of the assumed absence of defects in the layering. A further constraint follows from the assumed incompressibility, the velocity **v** therefore satisfying

$$\mathrm{div}\,\mathbf{v} = 0, \tag{2.2}$$

with the density ρ consequently a constant.

In Cartesian tensor notation the balance laws representing conservation of linear and angular momentum are

$$\rho\dot{v}_i = \rho F_i + t_{ij,j} \quad , \quad \rho K_i + e_{ijk}t_{kj} + \ell_{ij,j} = 0, \tag{2.3}$$

where **F** and **K** represent body forces and moments per unit mass, and **t** and ℓ the stress and couple stress tensors, respectively. The superposed dot denotes a material time derivative, and e_{ijk} the alternator. The inertial term is omitted from the second equation since it is generally considered negligible, and thermal effects are ignored.

The stress and couple stress tensors take the forms

$$\left.\begin{aligned} t_{ij} &= -p\delta_{ij} + \beta_p e_{pjk} a_{k,i} - \frac{\partial W}{\partial a_{k,j}} a_{k,i} - \frac{\partial W}{\partial c_{k,j}} c_{k,i} + \tilde{t}_{ij}, \\ \ell_{ij} &= \beta_p a_p \delta_{ij} - \beta_i a_j + e_{ipq}\left(a_p \frac{\partial W}{\partial a_{q,j}} + c_p \frac{\partial W}{\partial c_{q,j}}\right), \end{aligned}\right\} \tag{2.4}$$

respectively, where the pressure p and the vector $\boldsymbol{\beta}$ are consequences of the constraints (2.2) and the last of (2.1). The energy W is given by[2,3]

$$\begin{aligned} 2W &= K_1^a(a_{i,i})^2 + K_2^a(c_i a_{i,j} c_j)^2 + 2K_3^a a_{i,i} c_j a_{j,k} c_k \\ &+ K_1^c(c_{i,i})^2 + K_2^c c_{i,j} c_{i,j} + K_3^c c_{i,p} c_p c_{i,q} c_q + 2K_4^c c_{i,p} c_p c_{i,q} a_q \\ &+ 2K_1^{ac} c_{i,i} c_j a_{j,k} c_k + 2K_2^{ac} c_{i,i} a_{j,j} + 2K_5^c e_{ipq} c_p a_q c_{i,k} a_k, \end{aligned} \tag{2.5}$$

the Ks simply constants and the last term present only for chiral materials. The viscous stress tensor $\tilde{\mathbf{t}}$ is the sum of a symmetric part

$$\begin{aligned} \tilde{t}_{ij}^s &= \mu_o D_{ij} + \mu_1 a_p a_q D_{pq} a_i a_j + \mu_2(D_{ik} a_k a_j + D_{jk} a_k a_i) + \mu_3 c_p c_q D_{pq} c_i c_j \\ &+ \mu_4(D_{ik} c_k c_j + D_{jk} c_k c_i) + \mu_5 c_p a_q D_{pq}(c_i a_j + c_j a_i) + \lambda_1(A_i a_j + A_j a_i) \\ &+ \lambda_2(C_i c_j + C_j c_i) + \lambda_3 c_p A_p(c_i a_j + c_j a_i) \\ &+ \kappa_1(D_{ik} a_k c_j + D_{jk} a_k c_i + D_{ik} c_k a_j + D_{jk} c_k a_i) \\ &+ \kappa_2[2a_p c_q D_{pq} a_i a_j + a_p a_q D_{pq}(c_i a_j + c_j a_i)] \\ &+ \kappa_3[2a_p c_q D_{pq} c_i c_j + c_p c_q D_{pq}(c_i a_j + c_j a_i)] \\ &+ \tau_1(C_i a_j + C_j a_i) + \tau_2(A_i c_j + A_j c_i) + 2\tau_3 c_p A_p a_i a_j + 2\tau_4 c_p A_p c_i c_j, \end{aligned} \tag{2.6}$$

and a skew-symmetric part

$$\begin{aligned} \tilde{t}_{ij}^{ss} &= \lambda_1(D_{jk} a_k a_i - D_{ik} a_k a_j) + \lambda_2(D_{jk} c_k c_i - D_{ik} c_k c_j) \\ &+ \lambda_3 c_p a_q D_{pq}(a_i c_j - a_j c_i) + \lambda_4(A_j a_i - A_i a_j) \\ &+ \lambda_5(C_j c_i - C_i c_j) + \lambda_6 c_p A_p(a_i c_j - a_j c_i) + \tau_1(D_{jk} a_k c_i - D_{ik} a_k c_j) \\ &+ \tau_2(D_{jk} c_k a_i - D_{ik} c_k a_j) + \tau_3 a_p a_q D_{pq}(a_i c_j - a_j c_i) \\ &+ \tau_4 c_p c_q D_{pq}(a_i c_j - a_j c_i) + \tau_5(A_j c_i - A_i c_j + C_j a_i - C_i a_j), \end{aligned} \tag{2.7}$$

where

$$2D_{ij} = v_{i,j} + v_{j,i}\ ,\ 2W_{ij} = v_{i,j} - v_{j,i}\ ,\ A_i = \dot{a}_i - W_{ij} a_j\ ,\ C_i = \dot{c}_i - W_{ij} c_j, \tag{2.8}$$

and the coefficients are constants.

The body moments arising from external magnetic and electric fields can be expressed as

$$\rho K_i = e_{ijk}(a_j G^a_k + c_j G^c_k), \tag{2.9}$$

and equally the intrinsic viscous moment in the second of equations (2.3) can be written as

$$e_{ijk}\tilde{t}^{ss}_{kj} = e_{ijk}(a_j \tilde{g}^a_k + c_j \tilde{g}^c_k), \tag{2.10}$$

where

$$\left.\begin{aligned} \tilde{g}^a_i &= -2(\lambda_1 D_{ik} a_k + \lambda_3 c_i c_p a_q D_{pq} + \lambda_4 A_i + \lambda_6 c_i c_p A_p \\ &\quad \tau_2 D_{ik} c_k + \tau_3 c_i a_p a_q D_{pq} + \tau_4 c_i c_p c_q D_{pq} + \tau_5 C_i), \\ \tilde{g}^c_i &= -2(\lambda_2 D_{ik} c_k + \lambda_5 C_i + \tau_1 D_{ik} a_k + \tau_5 A_i), \end{aligned}\right\} \tag{2.11}$$

but we do not give the forms for $\mathbf{G}^a$ and $\mathbf{G}^c$ here, since they are not required. However, employing (2.9) and (2.10) the equations (2.3) can be recast as[1]

$$\left.\begin{aligned} \left(\frac{\partial W}{\partial a_{i,j}}\right)_{,j} - \frac{\partial W}{\partial a_i} + G^a_i + \tilde{g}^a_i + \gamma a_i + \mu c_i + e_{ijk}\beta_{k,j} = 0, \\ \left(\frac{\partial W}{\partial c_{i,j}}\right)_{,j} - \frac{\partial W}{\partial c_i} + G^c_i + \tilde{g}^c_i + \tau c_i + \mu a_i = 0, \end{aligned}\right\} \tag{2.12}$$

representing angular momentum balance, γ, μ and τ arbitrary scalars, and

$$\rho \dot{v}_i = \rho F_i - \tilde{p}_{,i} + \tilde{g}^a_k a_{k,i} + \tilde{g}^c_k c_{k,i} + \tilde{t}_{ij,j} \tag{2.13}$$

where

$$\tilde{p} = p + W + \psi \tag{2.14}$$

and ψ denotes the energy associated with the electric or magnetic field.

3. A SHEAR FLOW INSTABILITY

In this section we consider a smectic C liquid crystal confined between parallel plates with the smectic layers everywhere parallel to the plates, and subjected to a simple shear flow. In this geometry the relevant equations of the previous section have two solutions in which the alignment is uniform within the plane of shear. One such alignment proves to be stable to small perturbations while the other is unstable.[1] Consequently, if the smectic is initially aligned uniformly in the unstable configuration, one anticipates that a flow instability will occur in shear once the destabilising viscous torques exceed the elastic restoring torques. Below we present a brief analysis of this flow induced instability, which is somewhat analogous to the Pieranski-Guyon instability that occurs in a nematic.[8]

Choosing Cartesian axes with the z-axis normal to the plates and the x-axis parallel to the imposed flow, it is natural to seek solutions of the equations of the previous section of the form

$$\mathbf{a} = (0,0,1)\ ,\ \mathbf{c} = (\cos\phi(z), \sin\phi(z), 0)\ ,\ \mathbf{v} = (u(z), v(z), 0), \tag{3.1}$$

with the pressure p, the vector $\boldsymbol{\beta}$ and the multipliers γ, μ and τ also functions only of z. This choice is clearly consistent with the constraints (2.1). In this event the equations (2.12) reduce to

$$K_2^c \frac{d^2\phi}{dz^2} + (\tau_1 - \tau_5)(\sin\phi \frac{du}{dz} - \cos\phi \frac{dv}{dz}) = 0, \tag{3.2}$$

the remaining equations merely determining $\boldsymbol{\beta}, \gamma, \mu$ and τ, and equations (2.13) yield an expression for the pressure p and

$$\left.\begin{aligned} \tilde{t}_{xz} &= (\eta_1 + \eta_2 \cos^2\phi)\frac{du}{dz} + \eta_2 \sin\phi\cos\phi\frac{dv}{dz} = a, \\ \tilde{t}_{yz} &= (\eta_1 + \eta_2 \sin^2\phi)\frac{dv}{dz} + \eta_2 \sin\phi\cos\phi\frac{du}{dz} = b, \end{aligned}\right\} \tag{3.3}$$

where

$$2\eta_1 = \mu_o + \mu_2 - 2\lambda_1 + \lambda_4\ \ ,\ \ 2\eta_2 = \mu_4 + \mu_5 + 2\lambda_2 - 2\lambda_3 + \lambda_5 + \lambda_6, \tag{3.4}$$

and a and b are arbitrary constants. With the origin midway between the two plates, the relevant boundary conditions are

$$u(d) = V \quad , \quad u(-d) = v(\pm d) = 0 \quad , \quad \phi(\pm d) = \phi_o, \tag{3.5}$$

where $2d$ denotes the layer thickness, V the velocity of the upper plate, and ϕ_o is either zero or π. Straightforwardly one can readily verify that the above equations have simple solutions in which

$$\phi = \phi_o \quad , \quad u = \frac{V(z+d)}{2d} \quad , \quad v = 0 \quad , \quad \tilde{t}_{xz} = (\eta_1 + \eta_2)\frac{V}{2d} \quad , \quad \tilde{t}_{yz} = 0. \tag{3.6}$$

As Leslie, Stewart and Nakagawa[1] discuss, the above solutions are stable with respect to small, time dependent perturbations of the angle ϕ if

$$\phi_o = 0 \;\text{ and }\; \tau_5 > \tau_1 \quad , \quad \text{or} \;\; \phi_o = \pi \;\text{ and }\; \tau_1 > \tau_5, \tag{3.7}$$

being unstable otherwise.

We now consider small perturbations of the solutions (3.6) of the form

$$\phi = \phi_o + \hat{\phi}(z) \quad , \quad u = \frac{V(z+d)}{2d} + \hat{u}(z) \quad , \quad v = \hat{v}(z), \tag{3.8}$$

and obtain the following equations for these perturbations

$$K_2^c \frac{d^2\hat{\phi}}{dz^2} + (\tau_1 - \tau_5)\cos\phi_o\Big(\frac{V\hat{\phi}}{2d} - \frac{d\hat{v}}{dz}\Big) = 0, \tag{3.9}$$

$$(\eta_1 + \eta_2)\frac{d\hat{u}}{dz} = \hat{a} \quad , \quad \eta_1\frac{d\hat{v}}{dz} + \eta_2\frac{V\hat{\phi}}{2d} = \hat{b}, \tag{3.10}$$

$\hat{a}$ and $\hat{b}$ denoting perturbations to the imposed shear stresses. The relevant boundary conditions are

$$\hat{u}(\pm d) = \hat{v}(\pm d) = \hat{\phi}(\pm d) = 0. \tag{3.11}$$

The first of equations (3.10) allied to the first of the above quickly implies that $\hat{u}$ is zero, and elimination of $\hat{v}$ from the remaining equations yields

$$\frac{d^2\hat{\phi}}{dz^2} + \frac{V(\eta_1+\eta_2)(\tau_1-\tau_5)\cos\phi_o}{2K_2^c d\eta_1}\Big(\hat{\phi} - \frac{2d\hat{b}}{V(\eta_1+\eta_2)}\Big) = 0. \tag{3.12}$$

If $(\tau_1 - \tau_5)\cos\phi_o$ is positive, this equation has the solution subject to (3.11)

$$\hat{\phi} = \frac{2d\hat{b}}{V(\eta_1 + \eta_2)}\left(1 - \frac{\cos\omega z}{\cos\omega d}\right) \quad , \quad \omega^2 = \frac{V(\eta_1 + \eta_2)(\tau_1 - \tau_5)\cos\phi_o}{2K_2^c d\eta_1}, \tag{3.13}$$

and the boundary conditions for $\hat{v}$ are satisfied if

$$\frac{n_2 V}{2d}\int_{-d}^{d}\hat{\phi}dz = 2d\hat{b} \quad \Rightarrow \quad \eta_2 \tan\omega d + \eta_1 \omega d = 0, \tag{3.14}$$

which clearly yields positive values for ω. However, if $(\tau_1 - \tau_5)\cos\phi_o$ is negative, the relevant solution is

$$\hat{\phi} = \frac{2d\hat{b}}{V(\eta_1 + \eta_2)}\left(1 - \frac{\cosh\omega z}{\cosh\omega d}\right) \quad , \quad \omega^2 = \frac{V(\eta_1 + \eta_2)(\tau_5 - \tau_1)\cos\phi_o}{2K_2^c d\eta_1}, \tag{3.15}$$

and the condition (3.14) is replaced by

$$\eta_2 \tanh\omega d + \eta_1 \omega d = 0, \tag{3.16}$$

which has no non-trivial roots, since $\eta_1 + \eta_2$ is necessarily positive. Consequently we predict an instability with respect to the particular perturbations for the unstable members of (3.6) at a critical velocity V_c given by

$$V_c = 2K_2^c \eta_1 \zeta^2 / d(\eta_1 + \eta_2)(\tau_1 - \tau_5)\cos\phi_o, \tag{3.17}$$

denoting the smallest positive root of the equation

$$\eta_1 x + \eta_2 \tan x = 0. \tag{3.15}$$

Presumably this threshold will differ for other more general perturbations, but it is perhaps rather premature to attempt more detailed analyses at this stage, particularly since strong anchoring may not be the most appropriate boundary condition. Finally, while the above analysis is primarily for smectic C liquid crystals, it is important to note that it is also valid for smectic C^* liquid crystals if the gapwidth $2d$ is sufficiently small.

Clearly one can consider other variations of the above in which a magnetic field is applied either as a stabilising or destabilising influence, the latter allowing the flow to play a stabilising role. Also one can consider other geometries such as the bookshelf geometry. Blake and Leslie[10] discuss some of these possibilities in a separate paper.

4. A BACKFLOW EFFECT

In this section we discuss some solutions of the continuum equations that illustrate the possibility of backflow effects in smectic C liquid crystals somewhat like those observed in nematics.[9] Here also it is convenient to choose the simpler planar geometry so that the smectic layers are again parallel to the bounding plates, with initially the alignment uniform due to strong anchoring at the plates. Application of a magnetic field parallel to the plates but perpendicular to the initial alignment will rotate this alignment around the layer normal, the distortion presumably symmetric about the plane midway between the plates. Below our aim is to determine the initial relaxation of this distorted alignment on the removal of the field.

As in the previous section we again choose Cartesian axes so that the z-axis is normal to the plates with the origin midway between them, the x-axis now coincident with the direction of the initial alignment of the $\mathbf{c}$ director. Here also it is natural to look for solutions of the form (3.1), but the dependence is now upon both z and time t. In this event, the relevant equations are

$$K_2^c \frac{\partial^2 \phi}{\partial z^2} - 2\lambda_5 \frac{\partial \phi}{\partial t} + (\tau_1 - \tau_5)(\sin\phi \frac{\partial u}{\partial z} - \cos\phi \frac{\partial v}{\partial z}) = 0, \tag{4.1a}$$

and

$$\begin{aligned} \rho\frac{\partial u}{\partial t} &= \frac{\partial}{\partial z}\left[(\eta_1+\eta_2\cos^2\phi)\frac{\partial u}{\partial z}+\eta_2\sin\phi\cos\phi\frac{\partial v}{\partial z}+(\tau_5-\tau_1)\sin\phi\frac{\partial\phi}{\partial t}\right], \\ \rho\frac{\partial v}{\partial t} &= \frac{\partial}{\partial z}\left[(\eta_1+\eta_2\sin^2\phi)\frac{\partial v}{\partial z}+\eta_2\sin\phi\cos\phi\frac{\partial u}{\partial z}+(\tau_1-\tau_5)\cos\phi\frac{\partial\phi}{\partial t}\right], \end{aligned} \tag{4.1b}$$

the viscosities η_1 and η_2 again given by the expressions (3.4). From the above it is evident that one must include both flow components to avoid an over-determinate system. The boundary and initial conditions are

$$\begin{aligned} u(\pm d) = v(\pm d) = \phi(\pm d) = 0, \\ u(z,0) = v(z,0) = 0 \quad , \quad \phi(z,0) = \phi_o(z) \quad , \quad |z| \le d, \end{aligned} \tag{4.2}$$

where $\phi_o(z)$ is a known even function of z.

Following the approach of Clark and Leslie[9] for a nematic, it is convenient to introduce the notation

$$\begin{aligned} a = (\tau_5-\tau_1)\sin\phi_o \quad , \quad b = (\tau_1-\tau_5)\cos\phi_o, \\ m = \eta_1+\eta_2\cos^2\phi_o \quad , \quad n = \eta_2\sin\phi_o\cos\phi_o \quad , \quad p = \eta_1+\eta_2\sin^2\phi_o, \end{aligned} \tag{4.3}$$

where ϕ_o is some constant angle associated with the initial alignment profile. Clearly for a strong field one might select ϕ_o to be $\pi/2$, since the initial value of ϕ is close to this value over much of the cell, but equally it can be some average value associated with the function $\phi_o(z)$. As an approximation to the equations (4.1), we therefore consider the linear system

$$\left.\begin{aligned} K_2^c\frac{\partial^2\phi}{\partial z^2} - 2\lambda_5\frac{\partial\phi}{\partial t} - a\frac{\partial u}{\partial z} - b\frac{\partial v}{\partial z} = 0, \\ \rho\frac{\partial u}{\partial t} = m\frac{\partial^2 u}{\partial z^2} + n\frac{\partial^2 v}{\partial z^2} + a\frac{\partial^2\phi}{\partial z\partial t}, \\ \rho\frac{\partial v}{\partial t} = n\frac{\partial^2 u}{\partial z^2} + p\frac{\partial^2 v}{\partial z^2} + b\frac{\partial^2\phi}{\partial z\partial t}. \end{aligned}\right\} \tag{4.4}$$

An inspection of the last pair of equations, however, suggests that the inertial terms are likely to be small compared with the others, and so hereafter we set ρ equal to zero. In this event, it follows from the last two equations that

$$(mp-n^2)\frac{\partial^2 u}{\partial z^2} = (nb-ap)\frac{\partial^2\phi}{\partial z\partial t} \quad , \quad (mp-n^2)\frac{\partial^2 v}{\partial z^2} = (na-bm)\frac{\partial^2\phi}{\partial z\partial t}, \tag{4.5}$$

and so from the first that

$$K_2^c \frac{\partial^3 \phi}{\partial z^3} = \xi \frac{\partial^2 \phi}{\partial z \partial t} \quad , \quad \xi = 2\lambda_5 + \frac{a(nb-ap)+b(na-bm)}{mp-n^2} = 2\lambda_5 - \frac{(\tau_5-\tau_1)^2}{\eta_1}. \quad (4.6)$$

Consideration of viscous dissipation for this problem readily shows that the parameter ξ is always positive.

To proceed it is useful to introduce non-dimensional variables as follows

$$k = \frac{K_2^c}{2\lambda_5} \quad , \quad \tilde{u} = \frac{du}{k} \quad , \quad \tilde{v} = \frac{dv}{k} \quad , \quad \tilde{z} = \frac{z}{d} \quad , \quad \tilde{t} = \frac{kt}{d^2}, \quad (4.7)$$

and the above equations become

$$\left.\begin{aligned} &\frac{\partial^2 \phi}{\partial \tilde{z}^2} - \frac{\partial \phi}{\partial \tilde{t}} - \frac{a}{2\lambda_5}\frac{\partial \tilde{u}}{\partial \tilde{z}} - \frac{b}{2\lambda_5}\frac{\partial \tilde{v}}{\partial \tilde{z}} = 0, \\ (mp-n^2)&\frac{\partial^2 \tilde{u}}{\partial \tilde{z}^2} = (nb-ap)\frac{\partial^2 \phi}{\partial \tilde{z}\partial \tilde{t}} \quad , \quad (mp-n^2)\frac{\partial^2 \tilde{v}}{\partial \tilde{z}^2} = (na-bm)\frac{\partial^2 \phi}{\partial \tilde{z}\partial \tilde{t}}, \\ &\frac{\partial^3 \phi}{\partial \tilde{z}^3} = \mu \frac{\partial^2 \phi}{\partial \tilde{z}\partial \tilde{t}} \quad , \quad \mu = \frac{\xi}{2\lambda_5} \quad , \quad 0 < \mu < 1. \end{aligned}\right\} \quad (4.8)$$

Seeking solutions of the form

$$\phi = \Phi(\tilde{z})e^{-q^2\tilde{t}/\mu} \quad , \quad \tilde{u} = U(\tilde{z})e^{-q^2\tilde{t}/\mu} \quad , \quad \tilde{v} = V(\tilde{z})e^{-q^2\tilde{t}/\mu}, \quad (4.9)$$

one quickly finds noting the boundary conditions (4.2) that

$$\begin{aligned} \Phi(\tilde{z}) = A(\cos q\tilde{z} - \cos q), \quad U(\tilde{z}) &= \frac{Aq(ap-bn)}{\mu(mp-n^2)}(\sin q\tilde{z} - \tilde{z}\sin q), \\ V(\tilde{z}) &= \frac{Aq(bm-an)}{\mu(mp-n^2)}(\sin q\tilde{z} - \tilde{z}\sin q), \end{aligned} \quad (4.10)$$

provided that

$$(1-\mu)\tan q = q. \quad (4.11)$$

Given that this last equation has infinitely many roots, superposition yields

$$\left.\begin{aligned} \phi &= \sum_{n=1}^{\infty} A_n(\cos q_n \tilde{z} - \cos q_n)e^{-q_n^2 \tilde{t}/\mu}, \\ \tilde{u} &= (ap - bn)/\mu(mp - n^2) \sum_{n=1}^{\alpha} q_n A_n(\sin q_n \tilde{z} - \tilde{z} \sin q_n)e^{-q_n^2 \tilde{t}/\mu}, \\ \tilde{v} &= (bm - an)/\mu(mp - n^2) \sum_{n=1}^{\infty} q_n A_n(\sin q_n \tilde{z} - \tilde{z} \sin q_n)e^{-q_n^2 \tilde{t}/\mu}. \end{aligned}\right\} \quad (4.12)$$

The initial condition on the alignment (4.2) leads to

$$\sum_{n=1}^{\infty} A_n(\cos q_n \tilde{z} - \cos q_n) = \phi_o(z), \quad (4.13)$$

from which it follows that

$$A_n = \frac{2q_n}{q_n - \sin q_n \cos q_n} \int_0^1 \phi_o(d\tilde{z}) \cos q_n \tilde{z} d\tilde{z}, \quad (4.14)$$

This noting that the necessary orthogonal function is $\cos q_m \tilde{z}$.

Blake and Leslie[11] discuss those solutions in greater detail computing specific solutions by choosing values for the necessary material parameters, some of these available from a comparison with light-scattering data.[12] They find that a significant kickback effect can occur as in a nematic. Also they consider such effects in the bookshelf goemetry although there is a lack of information concerning possible values for material parameters in that case, which rather inhibits specific computation. However, here also one should bear in mind that strong anchoring may not be the most appropriate boundary condition, but nonetheless the above analysis does indicate that flow may play a significant role in such relaxation.

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